

New shapes of the ρ -meson light-cone distribution amplitudes: how can they influence the $B \rightarrow \rho e \nu$ decay form factors

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ABSTRACT: We present new models of the ρ -meson leading-twist light-cone distribution amplitudes based on the QCD sum rule approach with nonlocal condensates. Their shapes differ noticeably from that known in the literature. The phenomenological consequences for physically important process $B \rightarrow \rho e \nu$ are discussed in the framework of the light-cone sum rules. The results are compared with those found recently by P.Ball and V.M.Braun (1997).

1. Introduction

The physics of B -decay is an attractive field both from theoretical and experimental point of view. Among other important problems, the extraction of the CKM matrix elements from experimental data has received much attention as these elements determine our fundamental knowledge of the Standard Model. In this context the semileptonic B -decays to light hadrons (π, ρ) were mentioned as a suitable tool to measure the $|V_{ub}|$ (cf., e.g., [1]). Recently, the CLEO collaboration has confirmed the first experimental measurements [2] of the branching ratio for $B \rightarrow \rho l \nu$ and has presented first results for the t -dependence of the form factors.

As usual in QCD, one may hope that for a quark mass heavy enough and/or large momentum transfer, the large scale introduced would determine a perturbative regime that presumably would simplify the physical picture. However, due to quark confinement and especially for a heavy-to-light transitions (like $b \rightarrow u$), the analysis inevitably involves the (nonperturbative) dynamics of the light degrees of freedom. Thus, to disentangle the properties of the heavy quark, one is forced carefully to separate perturbative and nonperturbative effects.

The method of QCD Sum Rules (SRs) seems

to be well suited for such separation [3]. The effects of nonperturbative long-distance dynamics are accumulated into *universal* objects like vacuum condensates and, more generally, in hadronic distribution amplitudes (DAs) and bilocal correlators (see, e.g. [4, 5, 1]). In principle, the standard QCD SR approach implies investigation of a suitable 3-point correlator [6], and usually the first few terms of the operator product expansion (OPE) are involved. For the case of $B \rightarrow \rho$ weak transitions such a program was performed in Refs.[7, 8].

However, the kinematic region of interest for the momentum transfer to the lepton pair is quite large: $0 < t < 20.3 \text{ GeV}^2$, and one can encounter specific problems of the approach. Indeed, in the region of maximum recoil to the final light meson (this corresponds to momentum transfers $t \approx 0$, *i.e.* far from the threshold $t^{th} \sim m_b^2$) the relevant OPE for the 3-point correlator becomes poorly convergent, the correction terms being proportional to the positive (and growing with dimension of the condensate) powers of the heavy quark mass [8, 1]. This situation is in full correspondence with the previously investigated transition form factor $\gamma^*(Q)\gamma^*(q) \rightarrow \pi^0$ in the kinematics $Q^2 \gg q^2 \geq 1 \text{ GeV}^2$ [9, 5].

In this case one has to sum up the OPE se-

ries which naturally amounts to the nonlocal condensates (NLC) [9, 10]. These objects enter into different SRs and may be extracted from the relevant analysis.

On the other hand, in the above-mentioned kinematics the light-cone region dominates, which corresponds to the heavy quark perturbative propagation. A possible remedy of the problems with 3-point SRs was offered within the Light Cone (LC) SR approach [4, 11, 8, 1]. In this case one deals with an amplitude in which the final hadron is already represented by its DAs of leading twists. In comparison to the previous approach this amounts to an effective summation of the above-mentioned OPE series with the price of introducing other nonperturbative quantities – the DAs of the light hadrons. These DAs are *universal* quantities, they enter as important ingredients into the “factorization” formalism [12] for any hard exclusive reactions involving hadrons.

In the remaining of this short talk we shall present new results concerning the leading twist 2 DAs for a longitudinally and transversely polarized ρ -meson obtained from QCD SRs with NLCs. We estimate the influence of the new shapes of the DAs for the phenomenologically important weak form factors of the $B \rightarrow \rho$ transition using the LC SR in the leading twist approximation [8, 1].

2. The leading twist DAs of ρ -meson

Here, we discuss the light-cone DAs of the leading twist for the ρ -meson. At least for the leading twist DAs, a physical quark-parton interpretation exists: it is a nonperturbative amplitude for a hadron to decay into collinear quark(s)-antiquark(gluon).

The DAs under consideration, $\varphi_\rho^L(x)$, $\varphi_\rho^T(x)$, parameterize the gauge-invariant matrix elements with the $\rho(770)$ -meson ($J^{PC} = 1^{--}$) of the (non-local) vector current (μ^2 is the factorization scale),

$$\begin{aligned} \langle 0 | \bar{u}(z)E(z,0)\gamma_\mu d(0) | \rho_L(p) \rangle \Big|_{z^2=0} &= \\ &= if_\rho^L p_\mu \int_0^1 dx e^{ix(zp)} \varphi_\rho^L(x, \mu^2), \end{aligned}$$

and¹ the tensor current

$$\begin{aligned} \langle 0 | \bar{u}(z)E(z,0)\sigma_{\mu\nu}d(0) | \rho_\perp(p) \rangle \Big|_{z^2=0} &= \\ &= if_\rho^T (\varepsilon_\mu^\perp p_\nu - \varepsilon_\nu^\perp p_\mu) \int_0^1 dx e^{ix(zp)} \varphi_\rho^T(x, \mu^2), \end{aligned}$$

The first estimates of the nontrivial moments, $\langle \xi^N \rangle \equiv \int_0^1 \varphi(x)(2x-1)^N dx$, of these functions were obtained by Chernyak&Zhitnitsky (CZ) [13] using the QCD SR for suitable 2-point current correlators of the vector (tensor) currents with the derivatives. A detailed revision of these results within the standard approach were presented by Ball&Braun (BB)[14]. The analysis was also extended by introducing the DAs of nonleading twist (3, 4) and incorporating equations of motion (see, e.g., [8, 5]). In recent papers [15, 16], this so-called standard analysis was completed by taking into account the finite mass corrections as well. Note, that in the framework of the approach one should restrict oneself to an estimate of the 2-nd moment $\langle \xi^2 \rangle$ of the DA to restore its shape².

We would like to emphasize that the standard QCD SR approach for the nontrivial moments of the DAs encounters similar problems as mentioned above in the case of 3-point SRs. The relevant OPE for the N -th moment receives an N power enhancement, and the higher is the dimension of the operators involved in the OPE the stronger power growth is observed. Thus, the OPE for higher moments is poorly convergent and the evaluation of the moments hardly make sense (see the criticism in [9, 5, 17]).

It was recognized [10] that such an N enhancement is a consequence of expanding the originally NLCs, like $\langle \bar{q}(0)E(0,z)q(z) \rangle$, into the local ones $\langle \bar{q}(0)q(0) \rangle$, $\langle \bar{q}(0)\nabla^2q(0) \rangle$, etc., appearing in OPE. Physically this means that the correlation length of the vacuum fluctuations, λ_q^{-1} , was supposed to be much larger than the typical hadronic scale $\sim m_\rho^{-1}$ which appears to be an unrealistic approximation. On the contrary, keeping the NLCs unexpanded one would obtain a decreasing N dependence for the condensate

¹For $p_z \rightarrow \infty$ we incorporated that $\varepsilon_\mu^{\lambda=0} \simeq ip_\mu/m_\rho$.

²We should remark in this respect that the standard approach could not provide a reliable estimate even for the 2-nd moment of DA, see [10, 17, 18]

contributions just as it is the case with the leading perturbative term of the OPE [10]. On the basis of a simple model for NLC the authors of Ref. [10] reanalyzed the moment's QCD SR for the pion leading twist DA and obtained a form which is rather close to the asymptotic one $\varphi^{as}(x) = 6x(1-x)$. This result was in contrast to the double-humped form originally suggested by CZ. The closeness of the pion DA to its asymptotic form at a low normalization point was supported later using different theoretical approaches [5, 19, 20] and also from the experiment [21]. Here we present results for the two leading twist DAs of the ρ -meson using the same method and essentially the same models for the nonlocal condensates involved. Instead of going to details, we just briefly mention some essential features of the (NLC) QCD SRs for the relevant quantities.

The first five moments $N = 2, 4, 6, 8, 10$ have been obtained with a good accuracy for the DA of the longitudinally polarized ρ -meson [17]. The shape of the DA, $\varphi_\rho^L(x, \mu^2)$, restored with these moments (at $\mu^2 \simeq 1 \text{ GeV}^2$), is well established:

$$\begin{aligned} \varphi_\rho^L(x) = & \varphi^{as}(x) \times \quad (2.1) \\ & \times \left(1 + 0.077 C_2^{3/2}(\xi) - 0.077 C_4^{3/2}(\xi) \right), \end{aligned}$$

where $\xi \equiv 1 - 2x$, and $C_n^\nu(\xi)$ are the Gegenbauer polynomials. It does not differ strongly from that, obtained in the standard way [14], on the basis of a crude estimate of the second moment only. Nevertheless, one may observe an essential difference in the end-point behavior, numerically revealing itself in the important inverse moment of DA: $\int_0^1 \frac{\varphi_\rho^L(x)}{x} dx = 3$ (here), 3.54 (B&B), 4.38 (C&Z).

The case of transversally polarized ρ -meson is more peculiar because the tensor current is of mixed P-parity and projects also on states with $J^{PC} = 1^{+-}$ (the lowest resonance being the $b_1(1235)$ -meson). The relevant correlator of two tensor currents $\Pi_N^{\mu\nu;\alpha\beta}(q)$ contains different invariant form factors at the corresponding independent tensor structures, which, in general, can contaminate contributions from both the types of hadronic states.

In fact, in Refs. [13, 14], a mixed-parity SR was investigated based on the projection over

$z^\nu z^\beta g^{\mu\alpha}$. The feature of this SR is that the contribution of the four-quark condensate is absent. On the other hand, this SR receives a numerically strong contribution from the gluon condensate and, in fact, it should be sensitive to the model of the nonlocal entity, that is still ill-known, contrary to the quark case.

Thus, it is suggested to use, instead, a pure parity SR which relates only to states of definite parity (ρ , ρ' , etc. as $P = -1$). Such NLC SR allows one to extract not only tensor coupling f_ρ^T [14], but also the higher moments for $\langle \xi^N \rangle_\rho^T$ with $N = 2, 4, 6, 8$. It should be noted that contrary to the longitudinal case, the higher moments are far from their asymptotic values. The model for the DA $\varphi_\rho^T(x, \mu^2 \simeq 1 \text{ GeV}^2)$ reads:

$$\begin{aligned} \varphi_\rho^T(x) = & \varphi^{as}(x) \times \left(1 + 0.29 C_2^{3/2}(\xi) + \quad (2.2) \right. \\ & \left. + 0.41 C_4^{3/2}(\xi) - 0.32 C_6^{3/2}(\xi) \right), \end{aligned}$$

In Fig. 1, we have plotted our DA $\varphi_\rho^T(x)$ in comparison with that proposed by Ball&Braun [14]. One may observe an essential difference in the shape and especially for the end-point behavior.

The oscillatory form of our model DA is not an artifact of a by hand truncation of the series in Gegenbauer polynomials. In fact, using the higher moments obtained, we are able to calculate also the nonperturbative coefficients of the higher polynomial(s), which occurred to be very small. It is worth mentioning that the better knowledge of the end-point region in our NLC

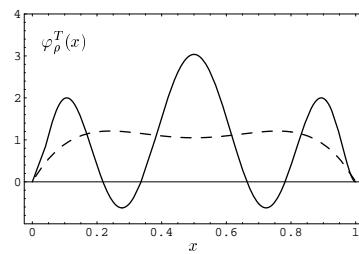


Figure 1: $\varphi_\rho^{T,mod}(x, 1 \text{ GeV}^2)$: our model – solid line, B&B model – dashed line.

SR approach is a consequence of the ability to extract the higher moments with enough good accuracy. Such a feature was, actually, expected because with the NLCs at hand our knowledge on the OPE side of the SR increases. Correspond-

ingly, one also may be able to extract a more detailed information on the phenomenological side. In fact, the mass $m_{\rho'}$, the decay constant(s) $f_{\rho'}^L$, $f_{\rho'}^T$, and the first 4 moments for the radially excited ρ'^L was obtained (cf. [17] for details).

3. The $B \rightarrow \rho e \nu$ transition form factors within the light-cone SR

The relevant invariant form factors of the independent Lorentz structures corresponding to the transition matrix element $\langle \rho, \lambda | (V - A)_\mu | B \rangle$ are denoted as $V(t)$, $A_1(t)$, and $A_2(t)$. As mentioned in the Introduction, the LC SR were proved to be a suitable approach to the semileptonic transition form factors, especially for the region of maximum recoil [1]. The “theoretical” side of the LC SR can be expressed as a convolution of a short distance coefficient function $CF(m_b, t, p_B^2; x, \mu^2)$ corresponding to the propagation of the heavy quark and the leading twist ρ -meson DAs.

In principle, it may receive corrections, both perturbative and nonperturbative. The α_s -corrections to the hard part $CF(\dots)$ as well as the contributions of higher twist (3 and 4) 2- and 3-body DAs amplitudes were investigated in detail in Refs. [15, 16]. The latter also include the “kinematic” higher twist corrections due to finite ρ -meson mass [16]. Not going to a detailed discussion of these comprehensive works, we mention that as a net result the impact of the α_s -corrections is on the level of 5% for relatively small momentum transfers, and the contribution of the higher twists is at most 3% (cf. [16]).

Thus, to estimate the influence of the new nonperturbative input presented in the previous section, we have used the LC SR in the leading twist approximation (cf. [1]). Just as in the case of the LC expansion for the transition amplitude $\gamma^* \gamma \rightarrow \pi^0$, one might expect high sensitivity to the end-point behavior of the DAs as they enter into convolution integrals like $\int_0^1 dx \varphi(x)/x$.

However, there are some essential differences which effectively soften our expectations. First, the DAs also enter into the “phenomenological” side of the SR in the “continuum” contribution of higher excited states in the channel with B -meson quantum numbers. This, actually, is a

specific feature of any LC SR. By subtracting the “continuum” one actually obtains “infrared safe quantities” like $\int_\epsilon^1 dx \varphi(x)/x$ where $\epsilon \simeq (m_b^2 - t)/(s_0^B - t)$, $m_b \simeq 4.8$ GeV, and $s_0^B \simeq 34$ GeV² is the continuum threshold in the B -channel³ as defined from the 2-point QCD SRs for the B -meson decay constant f_B (see [22, 23]).

For $t \approx 0$, $\epsilon \simeq 0.5 - 0.6$ and the LC SR should not be so sensitive to the end-point region $x \sim 0$. Obviously, the end-point region becomes to be important for higher momentum transfers t . However, for $t \geq 20$ GeV² the LC expansion would hardly make sense.

The second factor which eventually decreases the importance of the end-point region is connected with the standard Borel transformation of the SR with respect to the virtuality of the B -meson current: $-p_B^2 \rightarrow M_B^2$. The corresponding hard part $CF(\dots)$ then produces a standard suppression exponent: $\exp(\bar{x}(t - m_b^2)/x M_b^2)$. Numerically, it occurred to be less important.

We have treated the LC SRs using the same input parameters and the same procedure of extracting the physical form factors as in Ref.[1]. However, if one tries to fix the onset of the “continuum” by hand to the value $s_0^B \simeq 34$ GeV² dictated by the 2-point SRs for f_B , one encounters *inadmissible* uncertainties in the determination of the form factors when using our new nonperturbative input DAs. To get a stable SR, one is forced to allow a higher value for the s_0^B parameter.

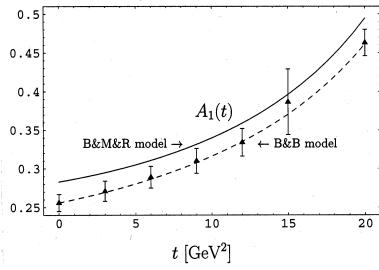
All evaluated form factors are a little bit larger than the corresponding estimations with the B&B leading twist DAs. The difference becomes more pronounced for large value of the momentum transfer t , ($m_b^2 - t \sim \mathcal{O}(m_b)$). The last is not surprising due to higher sensitivity to the end-point behavior of the input DA in this region. The form factors presented are determined with few times better processing accuracy with new “optimal” thresholds s_0^B . Note that the parameters of the usual “pole” parameterization of the form factors change significantly as compared

³As we shall see below, the LC SRs “prefer” a higher value.

to that in [1], *e.g.*,

$$A_1(t) = \frac{0.283}{1 - 0.157(t/m_B^2) - 0.837(t/m_B^2)^2}$$

The important form factor $A_1(t)$ (solid line) increases about 5 – 10% in comparison with the B&B result (the bars in the figure show the B&B errors), with an optimal threshold $s_0^B \simeq 45 \text{ GeV}^2$. From a physical point of view one should consider



the duality interval s_0^B as a characteristic of the spectra in the B -channel. Thus, in a self-consistent approach it is desirable to obtain the same (physical) value for s_0^B from different SRs.

However, the experimental information for higher excited states in the B -channel is poor [24]. From theoretical side, the value of f_B as well as s_0^B was a point of controversial issues (cf. [22, 23]). In the most detailed analysis of the 2-point SRs for f_B , the α_s -corrections to the leading term in the OPE were proven to be of importance [22, 23]. Actually, in Ref. [23], it was argued that an effective summation of the leading logs dictates the argument of α_s to be $\sim 1 \text{ GeV}$ rather than $\sim m_b$. As a result, the values of s_0^B from the interval $34 - 38 \text{ GeV}^2$ were preferred. In this context, the increase of the effective threshold s_0^B , as determined from the LC SR, demonstrates a deficiency of the Light Cone SRs for the $B \rightarrow \rho$ transition (at least, to the leading twist order).

We are grateful to V.M.Braun, H.G.Dosch, K.Goeke, M.Polyakov, N.G.Stefanis, and C.Weiss for useful discussions. This work was supported in part by the RFFI grant N 00-02-16696 and by the Heisenberg–Landau Program.

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